Language Issues in Mathematics

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Language Issues in Math

- Solutions to math problems need *words*, not just equations.
- Reasoning requires complete sentences.
- A big shift from high school math, where a sequence of formulas followed by a boxed answer may be OK.
- Students need to practice spoken and written mathematical expression: vocabulary, logic, notations, reasoning.

Language Issues

- Students may only know the logical connective "and": math is a big set of true statements only linked by "and".
- The logical "if/then" structure is crucial: one statement is true *because* another is, starting from definitions and axioms.
- There are also *procedures*, for computation or verifying statements, but those are secondary to reasoning.

Vocabulary

- Math has terminology for objects, processes, sets, logical relations, etc. Some is technical, much is everyday words with technical meanings. Some words have multiple meanings even within math.
- Degree, power, limit, integrate, diagonal, product, imaginary, orientation, continuous, absolute value, phase, order, closed, (in)finite, if and only if, linear, zero.
- Spherical triangle, vs. spherical container.
- Area of circle, vs. area of disk.
- "Define $f(x)$ by..." sounds like an instruction, but just provides information.

Vocabulary

- Additional issues for non-native English speakers/learners. Even British vs. American English!
- zed, naught, maths, indices, surd, HCF.
- Be aware of words you use or students struggle with.
- Students lack vocabulary for describing operations, reasoning. "Times it by 3". Hypothesis, conclusion, counterexample, lemma, weaker/stronger hypothesis.
- A chemistry Prof friend of mine did not know the word "differentiable".

Logical Expression

- If/then has many common meanings. Threats, cause/effect, evidence, counterfactual, etc. The mathematical meaning is close to *evidence*, but the treatment of false hypotheses is unique to math.
- If versus iff. Students often confuse hypothesis with conclusion, statements with their converses, or assume the conclusion midway through an argument. "If" is used in definitions but actually means "iff".
- Quantifiers present many challenges: multiply quantified statements, implicit quantifiers, ambiguous meanings of English expressions.
- Compare the use of "any": *Is any even number prime?* versus *True or false: Any even number is prime.* What about, $p^{1/n}$ *is irrational for any prime* p *and any integer* $n > 1$.

Quantifiers

- Statement: *The product of an irrational number with a nonzero rational number is irrational.* Proposed negation: *The product of an irrational number with a nonzero rational number is rational.* Proof by Contradiction by Example: $\pi \times 2$ is irrational, QED.
- Nested quantifiers: *For every* $x > 0$ *there exists* $y > 0$ *such that* $y < x$, versus *There exists* $y > 0$ *such that for every* $x > 0$, $y < x$. Variables must be chosen in order; later ones may depend on earlier ones.
- True/False: *A square is a rectangle.*